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Systematic Effects due to Noise in Interplanetary Scintillation Analysis

L. P. Pataki

Previous reports have shown how the three station data can be reduced to yield information on the interplanetary medium, the source of the second long L pulses. Briefly, correlations of amplitude are calculated, the three pairs of cross correlations are compared at an instant of time, and the shape of contours of equal correlation of amplitude are calculated through the use of the autocorrelation contour. These are related to the mean shape of the irregularities which cause the variations in amplitude (scintillations) by a Fourier transform. The effects of shape are then removed from the data by transforming to a system where the projection of the correlation contours is circular. Relative lags to peak cross correlation are then used to calculate cloud velocities. Examples of the procedure have been given in earlier reports.

A consideration of systematic errors which may be present led to a necessary refinement of the work. It is qualitatively obvious that the presence of random noise in the system must lower any measure of correlation which exists. Since only relative measures of correlation (cross correlations to each other and to autocorrelation) are used, this would not be a problem were it not for the fact that the noise is different on the different channels. In particular on 23 October 1965 the value of mean square noise for a piece of "quiet" record (i.e. no Jupiter emission in evidence) was .048 for the Bethany spaced receiver system, .157 for the Pomfret system, and .326 at Huntington in arbitrary but comparable units relative to mean square signal plus noise. Thus more careful treatment is needed.

With the assumption that the output of the system could be cast in the simple form

OUTPUT = SIGNAL + NOISE

an error analysis was carried through on the expected correlation (see appendix). It was indeed found that the noise is a critical factor in the experiment. Although it does not affect the shape of the correlation curves (except the autocorrelation near zero) and therefore does not affect the lag analysis, it does affect the numbers which go into the shape analysis which, as we noted, must be done prior to the lag portion of the analysis.

In accordance with the method outlined in the appendix the record of 23 October, 1965, was corrected for the effects of random noise. These results are shown in the attached graphs and tables. The shape analysis turned out to be relatively insensitive qualitatively, but the velocities depend critically on the exact quantitative results, and these were indeed affected. The original correlation ellipse had axial ratio 3.87 and an angle of inclination between the projection of Jupiter's vertical circle and the major axis of the ellipse of -48.6 degrees. The corresponding quantities for the noise corrected case are 4.73 and -45.4 degrees.

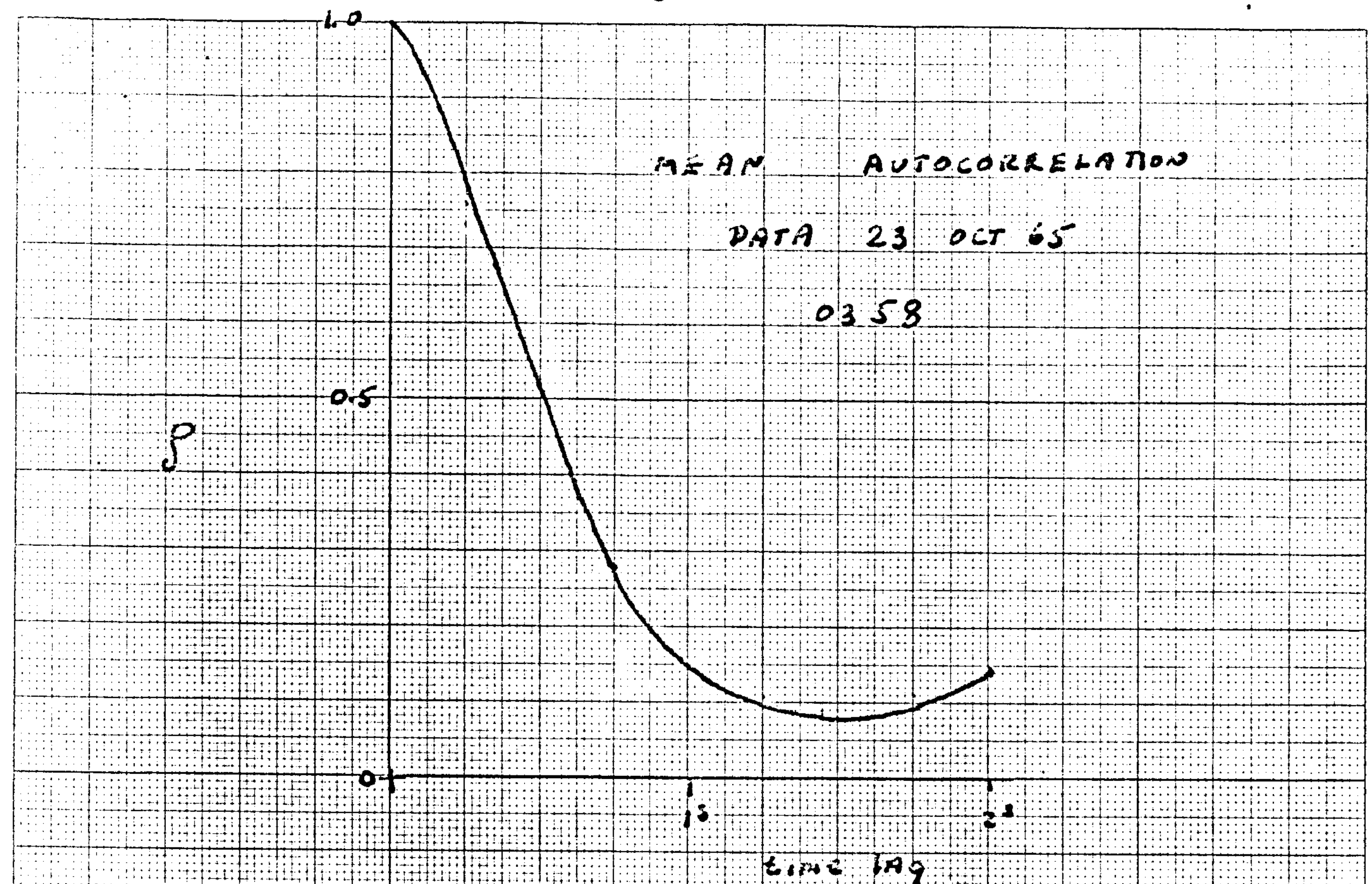
The geometric configuration derived from the original data gave an indeterminate result for the true drift velocity. The noise corrected data is consistent with a velocity of approximately 50 km/sec. along the ecliptic (within 10° of the ecliptic). The geometry, however, even in this case is such that this figure must be considered tenative pending the completion of further error analysis now in progress.

All records are now being processed to remove the effect noted above. Meanwhile theoretical calculations on the effects of correlated noise are proceeding. For example, we may consider the case

$$\text{OUTPUT} = \text{SIGNAL}\ (1 + \alpha) + \text{NOISE}$$

where α may be a multiplicative "noise" created by the ionosphere.

The removal of the systematic errors created by noise greatly enhances the faith we may put on the results of the study, and these corrections have become the first order of business.


MEAN AUTOCORRELATION
DATA 23 OCT 65
0358
1.0
0.5
ρ
0
1s
2s
time lag

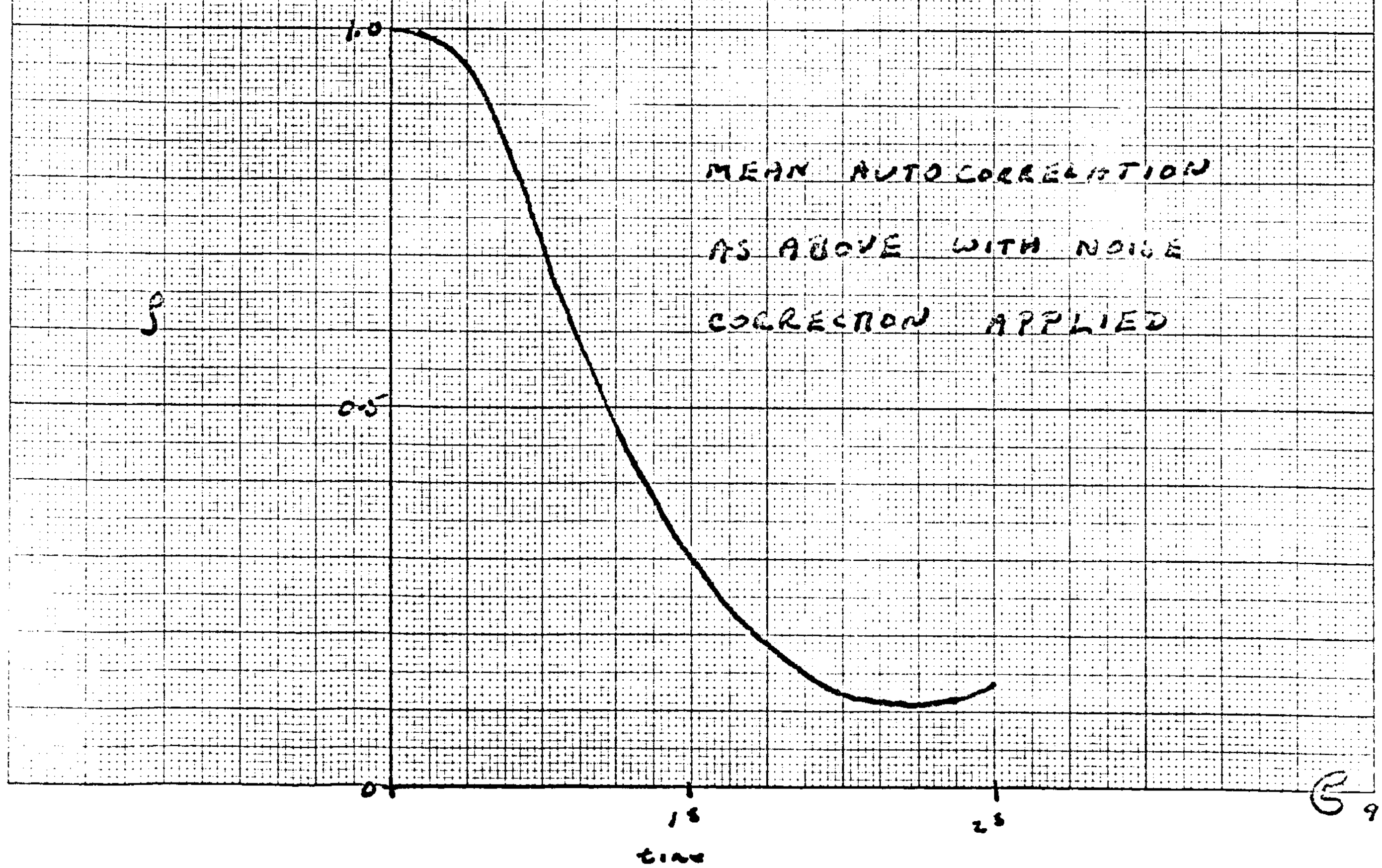

MEAN AUTOCORRELATION
AS ABOVE WITH NOISE
CORRECTION APPLIED
1.0
ρ
0.5
0
1s
2s
time

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Table I

Noise Correction Data of 23 Oct 1965

	BSR	PSR	HSR
$\Sigma m'$	219	392	367
$\overline{m'} = \Sigma m'/44$	4.977	8.91	8.34
$\overline{m'}^2$	24.77	79.39	69.6
$\Sigma m'^2$	1121	3595	3191
$\overline{m'^2}$	25.47	81.73	72.5
$\overline{m'^2} - \overline{m'}^2 \equiv \overline{m^2}$	0.70	2.34	2.9
$\sigma_{sig+noise}$	3.84	3.87	2.98
σ^2	14.74	14.98	8.88
$\frac{\overline{m^2}}{\sigma^2}$	0.048	0.156	0.326
$1 - \frac{\overline{m^2}}{\sigma^2}$	0.953	0.844	0.674
$1/1 - (\frac{\overline{m^2}}{\sigma^2})$	1.05	1.18	1.48

Headings refer to the three spaced receiver stations. Notation follows appendix. Primed noise refers to quantities before zero adjust.

The crosscorrelation correction factors are BSR-PSR 1.113, BSR-HSR 1.245, PSR-HSR 1.323.

Appendix

Errors in Correlation Coefficients

For two variables, $Z_i(t), Z_j(t)$, we define a correlation coefficient of arguement τ,

$$(1) \qquad \rho_{z_{ij}}(\tau) = \frac{\overline{z_i(t)\, z_j(t-\tau)} - \overline{z_i(t)}\;\overline{z_j(t)}}{\sigma_{z_i}\;\sigma_{z_j}}$$

Consider $y_i = x_i + n_i$ where y_i is observed, x_i is our desired signal, and n_i is the noise introdouced by the process of measurement both instrumental and human. We assume the noise introduced is random, in the sense of gaussian with zero mean, independent of the signal.

The elements of the correlation coefficient have the following properties:

$$\overline{y_i} = \overline{x_i + n_i} = \overline{x_i} + \overline{n_i} = \overline{x_i}$$

$$(2) \qquad \overline{x_i} = \overline{y_i}\,.$$

$$\overline{y_i(t)\, y_j(t-\tau)} = \overline{x_i(t) x_j(t-\tau) + n_i(t) n_j(t-\tau) + n_i(t)\, x_j(t-\tau) + n_j(t-\tau)\, x_i(t)}$$

$$= \overline{x_i(t)\, x_j(t-\tau)} + \overline{n_i(t)\, n_j(t-\tau)} + \left[\overline{n_i(t)\, x_j(t-\tau)} + \overline{n_j(t-\tau)\, x_i(t)}\right]$$

Note that as n_k and x_l are independent the term in [ ] is zero so that transposing we have

$$(3) \qquad \overline{x_i(t)\, x_j(t-\tau)} = \overline{y_i(t)\, y_j(t-\tau)} - \overline{n_i(t)\, n_j(t-\tau)}$$

Note that if $i \neq j$ the term $n_l(t)n_j(t-\tau)$ is zero while if $i=j$ it depends on the behavior of n_i as a function of time. In particular at $\tau = 0$ the term is $\overline{[n_i(t)]^2}$.

To examine the denominatör consider $\sigma_{y_i}^2$.

$$\sigma^2(y_i) = \sigma^2(x_i + n_i) = \sigma^2(x_i) + \sigma^2(n_i)\,.$$

However, as n_i has zero mean this last term is just $\overline{n_i^2}$ and we have

$$\sigma_{y_i}^2 = \sigma_{x_i}^2 + \overline{n_i^2}$$

or rearranging,

$$\sigma_{x_i}^2 = \sigma_{y_i}^2 \left[1 - \frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right]$$

and .

$$(4) \quad \sigma_{x_i} = \sigma_{y_i} \left[1 - \frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right]^{1/2}$$

note that $\sigma_{x_i}^2 \leq \sigma_{y_i}^2$ with equality only in the unphysical case of zero noise.

Substitution of equations (2), (3), and (4) into (1) yields

$$(5) \quad \rho_{x_{ij}}(\tau) = \frac{\overline{y_i(t)\, y_j(t-\tau)} - \overline{y_i(t)}\;\overline{y_j(t)} - \overline{n_i(t)\, n_j(t-\tau)}}{\sigma_{y_i}\, \sigma_{y_j} \left[\left(1 - \frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right)\left(1 - \frac{\overline{n_j^2}}{\sigma_{y_j}^2}\right)\right]^{1/2}}$$

We note the following special cases:

I. $i \neq j$

For this case $\overline{n_i n_j}$ is zero and

$$(6) \quad \rho_{x_{ij}}(\tau) = \frac{\overline{y_i(t)\, y_j(t-\tau)} - \overline{y_i(t)}\;\overline{y_j(t)}}{\sigma_{y_i}\, \sigma_{y_j} \left[\left(1 - \frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right)\left(1 - \frac{\overline{n_j^2}}{\sigma_{y_j}^2}\right)\right]^{1/2}} = \rho_{y_{ij}}(\tau) \left[\left(1 - \frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right)\left(1 - \frac{\overline{n_j^2}}{\sigma_{y_j}^2}\right)\right]^{-1/2}.$$

The term in square brackets is a constant independent of τ and the presence of noise decreases $\rho_{y_{ij}}(\tau)$ relative to $\rho_{x_{ij}}(\tau)$ by this constant factor.

II. $i = j$ For this case $\overline{n_i(t)\, n_i(t-\tau)}$ depends on the time behavior of n_i.

a) $\tau = 0$

$$\overline{n_i(t)\, n_i(t-\tau)} = \overline{[n_i(t)]^2} \equiv \overline{n_i^2}$$

thus we may rewrite (5) as

$$\rho_{x_{ii}}(0) = \frac{\overline{y_i(t)\, y_i(t)} - \overline{y_i(t)}\;\overline{y_i(t)} - \overline{n_i^2}}{\sigma_{y_i}^2 \left[1 - \frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right]} = \frac{\overline{y_i(t)^2} - \overline{y_i(t)}^2 - \overline{n_i^2}}{\sigma_{y_i}^2 \left[1 - \frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right]}$$

note $\sigma_{y_i}^2 = \overline{y_i(t)^2} - \overline{y_i(t)}^2$ thus

$$(7) \quad \rho_{x_{ii}}(0) = \frac{\sigma_{y_i}^2\left[1-\frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right]}{\sigma_{y_i}^2\left[1-\frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right]} = 1$$

and, as we expect, the noise does not affect this value.

b) $\tau \neq 0$.

In this case we must know the behavior of n_i as a function of t. We note, however, that at values of τ large enough $\overline{n_i(t)n_i(t-\tau)}$ is zero and we revert to the case given in (6). Note that the shape of $\rho_{y_{ii}}$ is not the same as $\rho_{x_{ii}}$ in the region $\tau \approx 0$. But we can recover $\rho_{x_{ii}}(\tau)$ by using (7) at $\tau=0$. and (6) at points where $\overline{n_i(t)n_i(t-\tau)} = 0$.

Special Conclusions for our Data

To obtain $\rho_{x_{ij}}(\tau)$ from $\rho_{y_{ij}}(\tau)$ we calculate

$$\left[\left(1-\frac{\overline{n_i^2}}{\sigma_{y_i}^2}\right)\left(1-\frac{\overline{n_j^2}}{\sigma_{y_j}^2}\right)\right]^{1/2}$$

and use equation (6).

To obtain $\rho_{x_{ii}}(\tau)$ from $\rho_{y_{ii}}(\tau)$ we use equation (6) when $\tau \geq 0^s.25$ with the bracket term taking the simple form $(1-\overline{n_i^2}/\sigma_{y_i}^2)$ and we note

$\rho_{x_{ii}}(0) = 1$.

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